

Markscheme

May 2022

**Mathematics:
applications and interpretation**

Standard level

Paper 2

16 pages

© International Baccalaureate Organization 2022

All rights reserved. No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without the prior written permission from the IB. Additionally, the license tied with this product prohibits use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, whether fee-covered or not, is prohibited and is a criminal offense.

More information on how to request written permission in the form of a license can be obtained from <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organisation du Baccalauréat International 2022

Tous droits réservés. Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite préalable de l'IB. De plus, la licence associée à ce produit interdit toute utilisation de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, moyennant paiement ou non, est interdite et constitue une infraction pénale.

Pour plus d'informations sur la procédure à suivre pour obtenir une autorisation écrite sous la forme d'une licence, rendez-vous à l'adresse <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

© Organización del Bachillerato Internacional, 2022

Todos los derechos reservados. No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin la previa autorización por escrito del IB. Además, la licencia vinculada a este producto prohíbe el uso de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales—, ya sea incluido en tasas o no, está prohibido y constituye un delito.

En este enlace encontrará más información sobre cómo solicitar una autorización por escrito en forma de licencia: <https://ibo.org/become-an-ib-school/ib-publishing/licensing/applying-for-a-license/>.

Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3**, **M2** etc., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award A1 for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award A0 for the final mark (and full FT is available in subsequent parts)

3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$.

An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^x$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate **A** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) (i) 0.58 (s) A1
 (ii) $0.7 - 0.42$ (A1)(M1)

Note: Award **A1** for correct quartiles seen, **M1** for subtraction of their quartiles.

0.28 s A1
[4 marks]

- (b) 9 (people have reaction time ≤ 0.4) (A1)
 31 (people have reaction time > 0.4) A1
[2 marks]

- (c) $(90\% \times 40 =) 36$ **OR** 4 (A1)
 0.8 s A1
[2 marks]

- (d) (i) $(a =) 6$ A1
 (ii) $(b =) 4$ A1
[2 marks]

- (e) $0.6 < t \leq 0.8$ A1
[1 mark]

- (f) 0.55 s A2
[2 marks]

- (g) the mean will increase A1
 because the incorrect reaction times are moving from a lower interval
 to a higher interval which will increase the numerator of the mean calculation R1

 the median will stay the same A1
 because the median or middle of the data is greater than both intervals
 being changed R1

Note: Do not award **A1R0**.

[4 marks]
Total [17 marks]

2. (a) (i) EITHER

$$115.5 = u_1 + (3 - 1) \times d \quad (115.5 = u_1 + 2d)$$

$$108 = u_1 + (8 - 1) \times d \quad (108 = u_1 + 7d) \quad (M1)(A1)$$

Note: Award **M1** for attempting to use the arithmetic sequence term formula, **A1** for both equations correct. Working for **M1** and **A1** can be found in parts (i) or (ii).

$$(d = -1.5)$$

1.5 (cups/day) A1

Note: Answer must be written as a positive value to award **A1**.

OR

$$(d =) \frac{115.5 - 108}{5} \quad (M1)(A1)$$

Note: Award **M1** for attempting a calculation using the difference between term 3 and term 8; **A1** for a correct substitution.

$$(d =) 1.5 \text{ (cups/day)} \quad A1$$

(ii) $(u_1 =) 118.5 \text{ (cups)}$ A1

[4 marks]

(b) attempting to substitute their values into the term formula for arithmetic sequence equated to zero (M1)

$$0 = 118.5 + (n - 1) \times (-1.5)$$

$$(n =) 80 \text{ days} \quad A1$$

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

(c) $(t_5 =) 625 \times 1.064^{(5-1)}$ (M1)(A1)

Note: Award **M1** for attempting to use the geometric sequence term formula; **A1** for a correct substitution

$$\text{\$ } 801 \quad A1$$

Note: The answer must be rounded to a whole number to award the final **A1**.

[3 marks]

continued...

Question 2 continued

(d) (i) ($S_{10} =$) (\$) 8390 (8394.39...) **A1**

(ii) **EITHER**

the total cost (of dog food) **R1**
for 10 years beginning in 2021 **OR** 10 years before 2031 **R1**

OR

the total cost (of dog food) **R1**
from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 **R1**

[3 marks]

(e) **EITHER**

According to the model, the cost of dog food per year will eventually be too high to keep a dog.

OR

The model does not necessarily consider changes in inflation rate.

OR

The model is appropriate as long as inflation increases at a similar rate.

OR

The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.

OR

The model is appropriate since dog food bags can only be bought in discrete quantities.

R1

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either “model” is mentioned specifically, or other mathematical terms such as “increasing” or “discrete quantities” are seen. Do not accept a contextual argument in isolation, e.g. “The dog will eventually die”.

[1 mark]

Total [13 marks]

3. (a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ (M1)
 (4, 1) A1

Note: Award **A0** if parentheses are omitted in the final answer.

[2 marks]

- (b) attempt to substitute values into gradient formula (M1)
 $\left(\frac{0-2}{6-2} = \right) -\frac{1}{2}$ (A1)
 therefore the gradient of perpendicular bisector is 2 (M1)
 so $y-1 = 2(x-4)$ ($y = 2x-7$) A1

[4 marks]

- (c) identifying the correct equations to use: (M1)
 $y = 2-x$ and $y = 2x-7$
 evidence of solving their correct equations or of finding intersection point graphically (M1)
 (3, -1) A1

Note: Accept an answer expressed as " $x = 3, y = -1$ ".

[3 marks]

- (d) attempt to use distance formula (M1)
 $YZ = \sqrt{(7-(-1))^2 + (7-3)^2}$
 $= \sqrt{80} (4\sqrt{5})$ A1

[2 marks]

- (e) **METHOD 1 (cosine rule)**
 length of XZ is $\sqrt{80} (4\sqrt{5}, 8.94427\dots)$ (A1)

Note: Accept 8.94 and 8.9.

- attempt to substitute into cosine rule (M1)
 $\cos \hat{X}YZ = \frac{80+32-80}{2 \times \sqrt{80} \sqrt{32}} (= 0.316227\dots)$ (A1)

Note: Award **A1** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

- $(\hat{X}YZ =) 71.6^\circ (71.5650\dots^\circ)$ A1

Note: Last **A1** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

continued...

Question 3 continued

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ ($4\sqrt{5}$, 8.94427...) **(A1)**

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ **(M1)(A1)**

Note: Award **A1** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule.
Exact values do not need to be used in the substitution.

$(\hat{X}\hat{Y}\hat{Z} =) 71.6^\circ$ (71.5650°) **A1**

Note: Last **A1** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\dots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ **(M1)**

= 24 km² **A1**
[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc.

R1
[1 mark]
Total [18 marks]

4. (a) (i) an attempt to find the amplitude (M1)
 $\frac{61.8}{2}$ **OR** $\frac{64.5-2.7}{2}$
 (a =) 30.9 m A1

Note: Accept an answer of (a =) -30.9 m.

- (ii) (period = $\frac{60}{1.5}$ =) 40 (s) (A1)
 ((b =) $\frac{360^\circ}{40}$)
 (b =) 9 A1

Note: Accept an answer of (b =) -9.

- (iii) attempt to find d (M1)
 (d =) $30.9 + 2.7$ **OR** $\frac{64.5 + 2.7}{2}$
 (d =) 33.6 m A1
[6 marks]

- (b) 12×1.5 **OR** $\frac{12 \times 60}{40}$ (M1)
 18 (revolutions per ride) A1
[2 marks]

- (c) (i) $0 \leq t \leq 720$ A1
 (ii) $2.7 \leq h \leq 64.5$ A1A1

Note: Award **A1** for correct endpoints of domain and **A1** for correct endpoints of range. Award **A1** for correct direction of both inequalities.

[3 marks]

- (d) graph of $h(t)$ and $y = 16.7$ **OR** $h(t) = 16.7$ (M1)
 6.31596... and 33.6840... (A1)
 27.4 (s) (27.3680...) A1
[3 marks]

continued...

Question 4 continued

(e) (i) d **A1**

(ii) **EITHER**
 $d + 30.9 = 65.2$ **(A1)**

OR
 $65.2 - (61.8 + 2.7) = 0.7$ **(A1)**

OR
3.4 (new platform height) **(A1)**

THEN
 $(d =) 34.3 \text{ m}$ **A1**

[3 marks]
Total [17 marks]

5. (a) attempt to expand given expression (M1)

$$C = \frac{xk^2}{10} - \frac{3x^3}{1000}$$

$$\frac{dC}{dx} = \frac{k^2}{10} - \frac{9x^2}{1000}$$

M1A1

Note: Award **M1** for power rule correctly applied to at least one term and **A1** for correct answer.

[3 marks]

- (b) equating their $\frac{dC}{dx}$ to zero (M1)

$$\frac{k^2}{10} - \frac{9x^2}{1000} = 0$$

$$x^2 = \frac{100k^2}{9}$$

$$x = \frac{10k}{3}$$

(A1)

substituting their x back into given expression

(M1)

$$C_{\max} = \frac{10k}{30} \left(k^2 - \frac{300k^2}{900} \right)$$

$$C_{\max} = \frac{2k^3}{9} (0.222\dots k^3)$$

A1

[4 marks]

- (c) (i) substituting 20 into given expression and equating to 426 M1

$$426 = \frac{20}{10} \left(k^2 - \frac{3}{100} (20)^2 \right)$$

$$k = 15$$

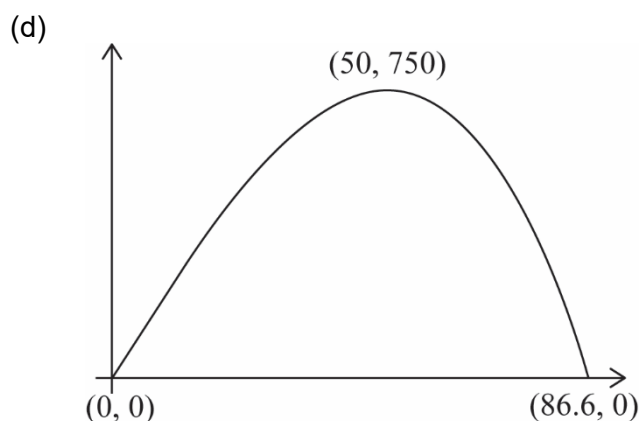
A1

- (ii) 50 A1

[3 marks]

continued...

Question 5 continued



A1A1A1

Note: Award **A1** for graph drawn for positive x indicating an increasing and then decreasing function, **A1** for maximum labelled and **A1** for graph passing through the origin and 86.6, marked on the x -axis or whose coordinates are given.

[3 marks]

(e) setting their expression for C to zero **OR** choosing correct x -intercept on their graph of C

$x_{\max} = 86.6$ (86.6025...) litres

(M1)

A1

[2 marks]

Total [15 marks]